

Soluzioni

18. pag. 16

[Ad esempio: $(4, 7)$; $(3, 6)$; $(\frac{5}{2}, \frac{11}{2})$; infiniti]





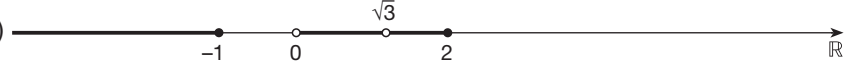
19. pag. 16

[Ad esempio: $(-3, 2)$ e $(1, 6)$; sì, se $a \neq b$]

20. pag. 16

[Sì, ad esempio: $(-\frac{7}{2}, \frac{5}{2})$; no; no]

21. pag. 17

- a) 1) 
- 2) *punti interni* $1 < x < 2 \vee x > 5$
punti di accumulazione $1 \leq x \leq 2 \vee x \geq 5$
punti isolati $x = 0$ $FE = \{0, 1, 2, 5\}$
- 3) E è un insieme né aperto né chiuso
- b) 1) 
- 2) *punti interni* $x < 1 - \sqrt{2} \vee x > 1 + \sqrt{2}$
punti di accumulazione $x \leq 1 - \sqrt{2} \vee x \geq 1 + \sqrt{2}$
punti isolati nessuno $FE = \{1 - \sqrt{2}, 1 + \sqrt{2}\}$
- 3) E è un insieme aperto
- c) 1) 
- 2) *punti interni* $0 < x < 3 \vee x > 7$
punti di accumulazione $0 \leq x \leq 3 \vee x \geq 7$
punti isolati $x = -\frac{1}{5} \vee x = 4$ $FE = \{-\frac{1}{5}, 0, 3, 4, 7\}$
- 3) E è un insieme né aperto né chiuso
- d) 1) 
- 2) *punti interni* $-1 < x < 1 \vee x > \sqrt{5}$
punti di accumulazione $-1 \leq x \leq 1 \vee x \geq \sqrt{5}$
punti isolati $x = 2$ $FE = \{-1, 1, 2, \sqrt{5}\}$
- 3) E è un insieme né aperto né chiuso
- e) 1) 
- 2) *punti interni* $x < -1 \vee 0 < x < \sqrt{3} \vee \sqrt{3} < x < 2$
punti di accumulazione $x \leq -1 \vee 0 \leq x \leq 2$
punti isolati nessuno $FE = \{-1, 0, \sqrt{3}, 2\}$
- 3) E è un insieme né aperto né chiuso

22. pag. 17

$$\left[\text{Ad esempio: } E = \left\{ x \in \mathbb{R} \mid x = \frac{1}{n}, n \in \mathbb{N}^* \right\} \right]$$

23. pag. 17

[Ad esempio: qualsiasi intervallo (limitato o illimitato) come
 $E = (-2, 6]$ oppure $E = (-\infty, \sqrt{2}]$]

24. pag. 17

[Tutti i punti di A ; $x = 3$; $x \in \mathbb{N} - \{0, 1, 2\}$]

25. pag. 17

$$\left[\text{Ad esempio: } E = [-2, -1) \cup \{0\} \cup \left(\frac{1}{2}, \sqrt{3} \right) \right]$$

26. pag. 17

[Ad esempio: $[-1, 0) \cup \{2, 3\}$ oppure lo stesso insieme dell'es. 25]

27. pag. 17

$$\left[x = \frac{1}{2} \vee x = 1 \right]$$

28. pag. 17

$$[x = 5]$$

29. pag. 17

[Ad esempio: $A = \{-2\} \cup (-1, 5]$ e $B = (5, 7] \cup (10, +\infty)$]

30. pag. 17

$$[-3 \leq x \leq 1; 2 \leq x \leq 7; 0 \leq x \leq 2]$$

1. pag. 30

$$\left[\frac{1}{4}; 1; 2; \frac{1}{8}; \frac{1}{4}; 1; \frac{1}{2}; 2^{|\alpha|-2}; 1 \right]$$

2. pag. 30

$$\left[0; \frac{\alpha^3 - 1}{\alpha}; \frac{\alpha(\alpha^2 + 3\alpha + 3)}{\alpha + 1}; \frac{\alpha^3 - 3\alpha^2 + 3\alpha - 2}{\alpha - 1}; \frac{8\alpha^3 - 1}{\alpha}; \frac{t}{t^3 - 1}; \frac{1 - t^3}{t^2} \right]$$

4. pag. 30

[a) no; b) no; c) sì; d) sì; e) no; f) sì; g) no; h) sì]

5. pag. 30

[a) algebriche razionali intere; b) algebriche razionali fratte; c) algebriche irrazionali;
 d), e) trascendenti logaritmiche; f) trascendenti esponenziali;
 g) trascendenti goniometriche; h) trascendenti di vario tipo]

6. pag. 30

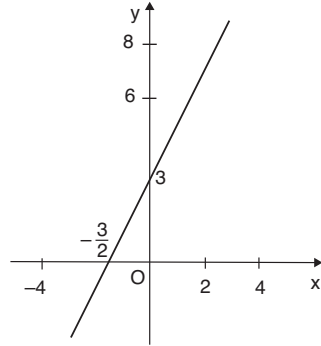
$$[y = x^3]$$

7. pag. 30

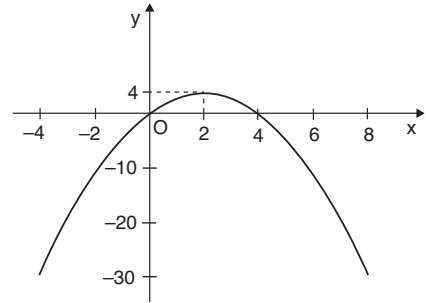
[Le prime due e l'ultima]

8. pag. 36 *Attenzione! Alcuni grafici sono in doppia scala.*

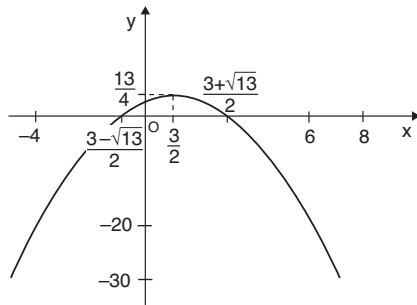
[a)



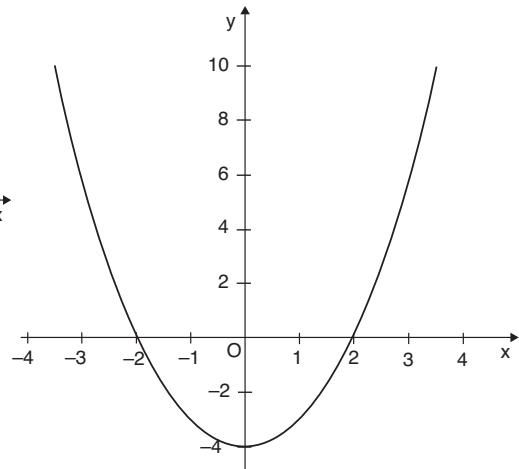
$$f(x) = 2x + 3$$



$$f(x) = 4x - x^2$$

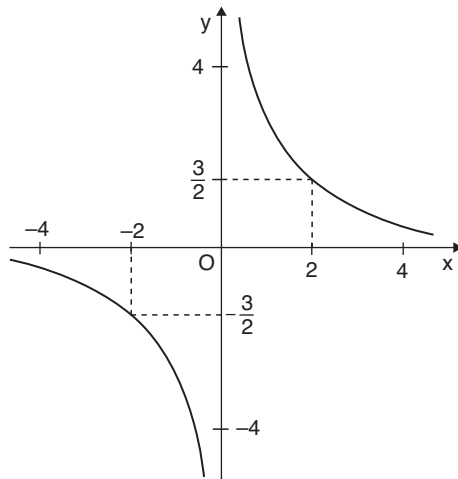


$$f(x) = 3x - x^2 + 1$$

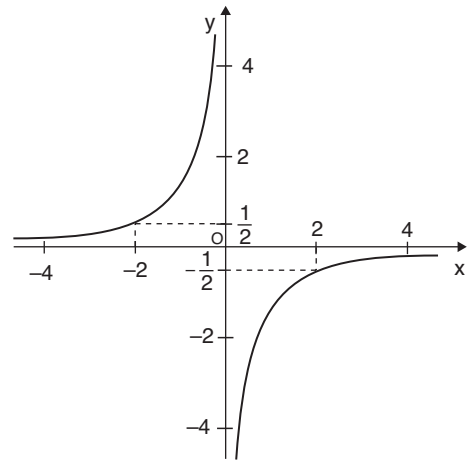


$$f(x) = x^2 - 4$$

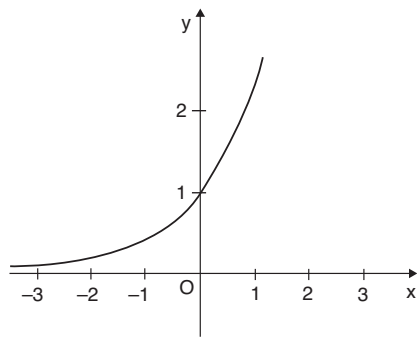
b)



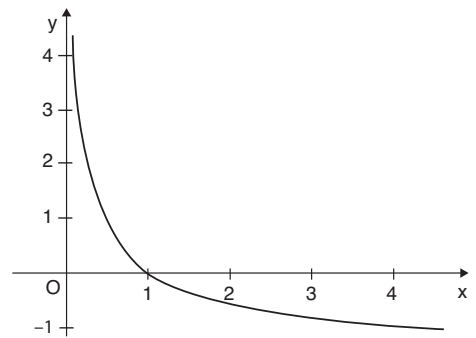
$$f(x) = \frac{3}{x}$$



$$f(x) = -\frac{1}{x}$$

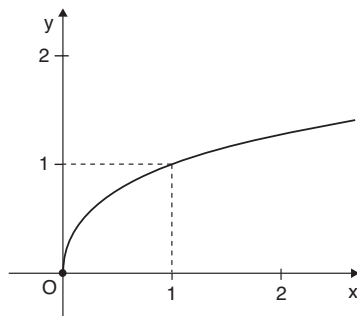


$$f(x) = 3^x$$

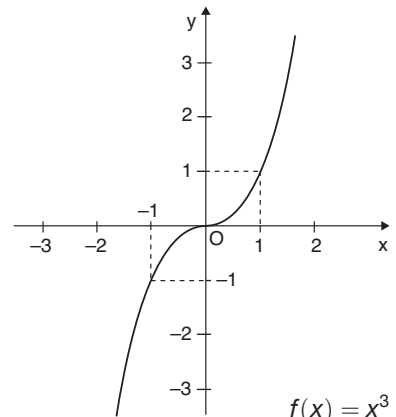


$$f(x) = \log_{\frac{1}{4}} x$$

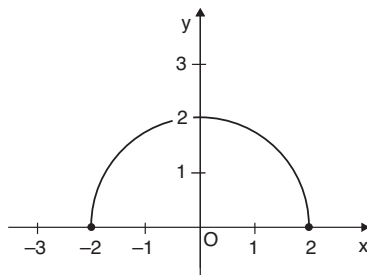
c)



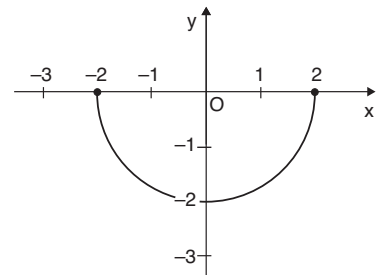
$$f(x) = \sqrt{x}$$



$$f(x) = x^3$$

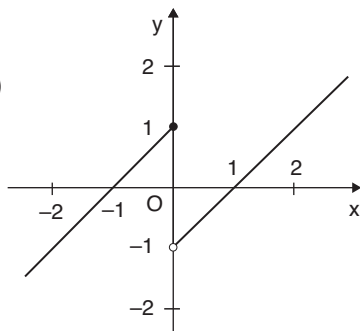


$$f(x) = \sqrt{4 - x^2}$$

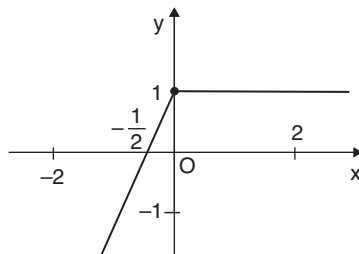


$$f(x) = -\sqrt{4 - x^2}$$

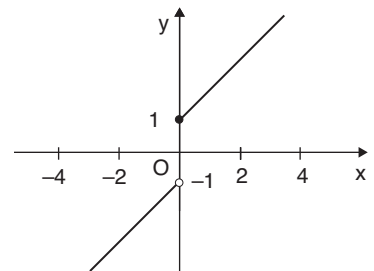
d)



$$f(x) = \begin{cases} x+1 & \text{per } x \leq 0 \\ x-1 & \text{per } x > 0 \end{cases}$$

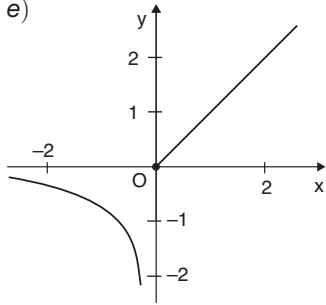


$$f(x) = \begin{cases} 2x+1 & \text{per } x \leq 0 \\ 1 & \text{per } x > 0 \end{cases}$$

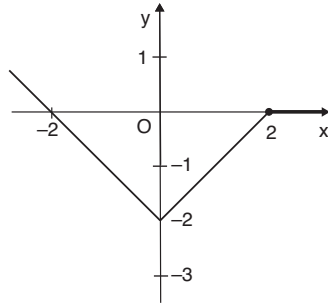


$$f(x) = \begin{cases} \sqrt{x^2} + x & \text{per } x \neq 0 \\ 1 & \text{per } x = 0 \end{cases}$$

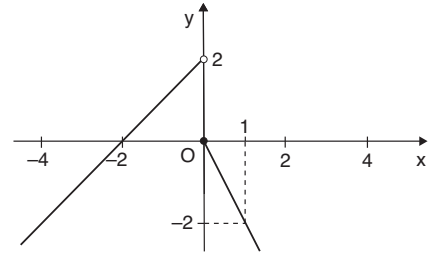
e)



$$f(x) = \begin{cases} \frac{1}{x} & \text{per } x < 0 \\ x & \text{per } x \geq 0 \end{cases}$$

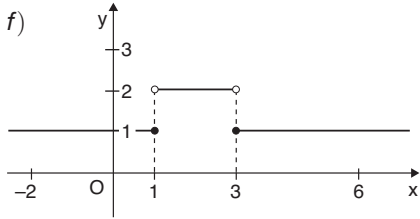


$$f(x) = \begin{cases} |x| - 2 & \text{per } x \leq 2 \\ 0 & \text{per } x > 2 \end{cases}$$

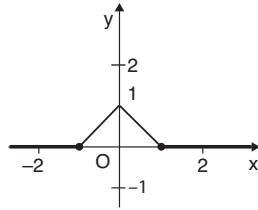


$$f(x) = \begin{cases} x + 2 & \text{per } x < 0 \\ 0 & \text{per } x = 0 \\ -2x & \text{per } x > 0 \end{cases}$$

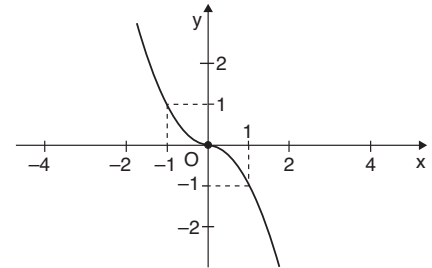
f)



$$f(x) = \begin{cases} 1 & \text{per } x \leq 1 \vee x \geq 3 \\ 2 & \text{per } 1 < x < 3 \end{cases}$$

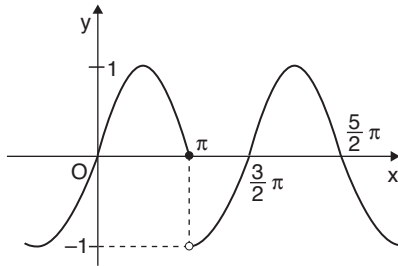


$$f(x) = \begin{cases} 1 - |x| & \text{per } |x| \leq 1 \\ 0 & \text{per } |x| > 1 \end{cases}$$

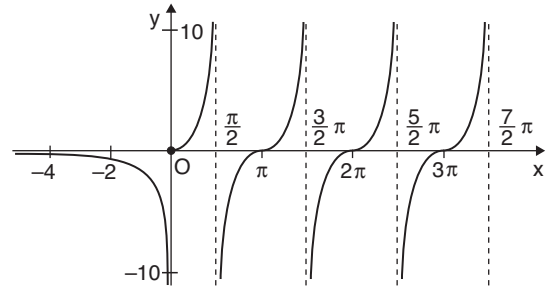


$$f(x) = \begin{cases} x^2 & \text{per } x \leq 0 \\ -x^2 & \text{per } x > 0 \end{cases}$$

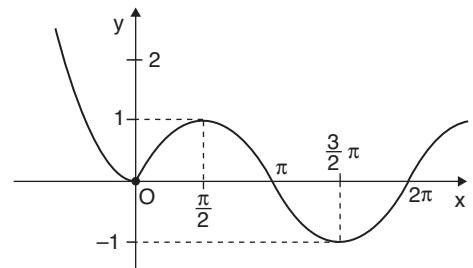
g)



$$f(x) = \begin{cases} \text{sen } x & \text{per } x \leq \pi \\ \text{cos } x & \text{per } x > \pi \end{cases}$$

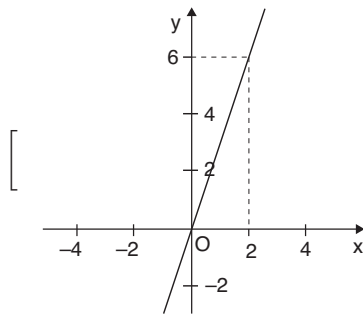


$$f(x) = \begin{cases} \frac{1}{x} & \text{per } x < 0 \\ \text{tg } x & \text{per } x \geq 0 \end{cases}$$

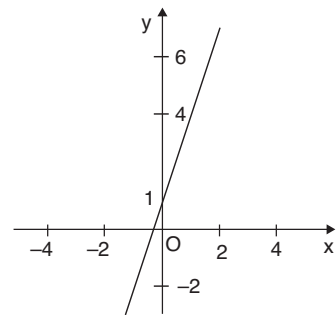


$$f(x) = \begin{cases} x^2 & \text{per } x \leq 0 \\ \text{sen } x & \text{per } x > 0 \end{cases} \quad]$$

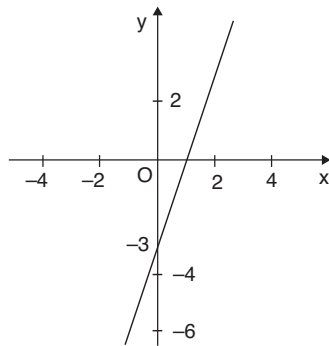
9. pag. 36



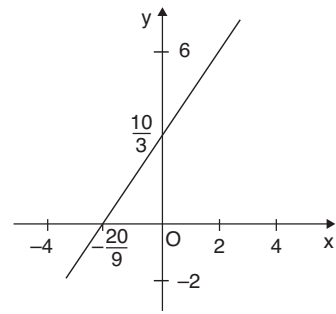
$$f(x) = 3x$$



$$f(x) = 3x + 1$$

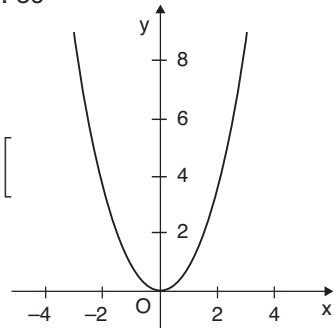


$$f(x) = 3(x - 1)$$

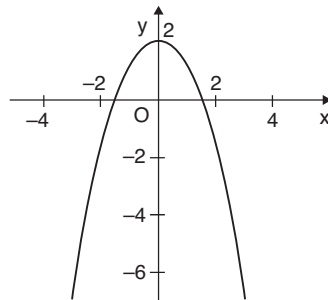


$$f(x) = \frac{3}{2}(x + 2) + \frac{1}{3}$$

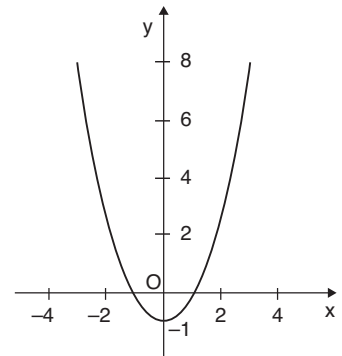
10. pag. 36



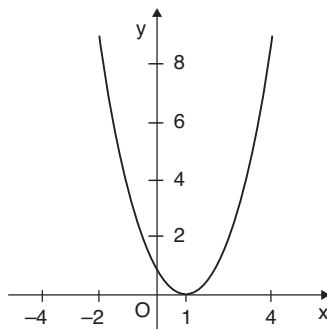
$$f(x) = x^2$$



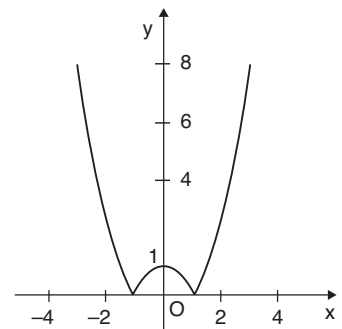
$$f(x) = -x^2 + 2$$



$$f(x) = x^2 - 1$$

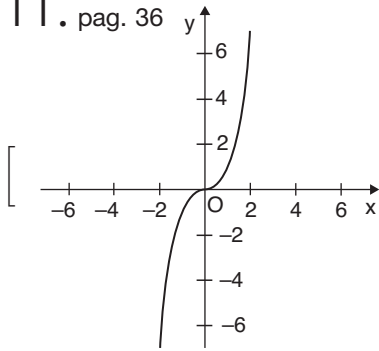


$$f(x) = (x - 1)^2$$

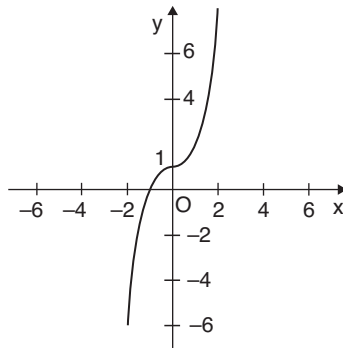


$$f(x) = |x^2 - 1|$$

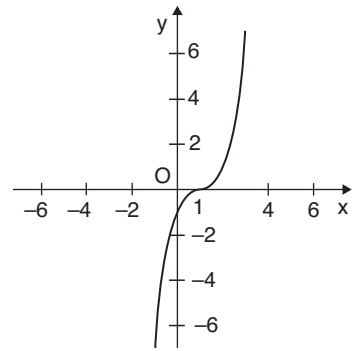
11. pag. 36



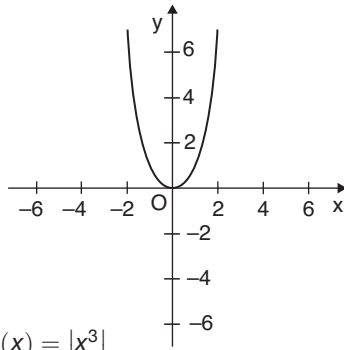
$$f(x) = x^3$$



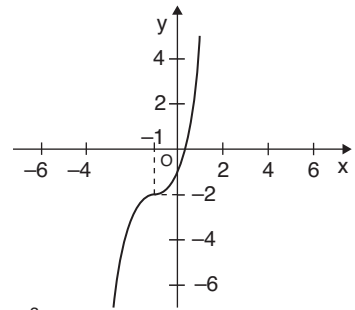
$$f(x) = x^3 + 1$$



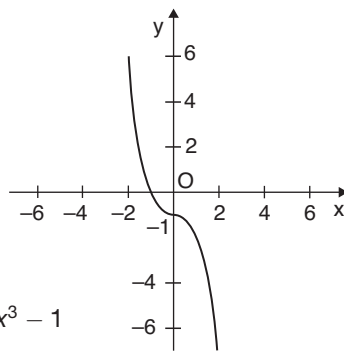
$$f(x) = (x-1)^3$$



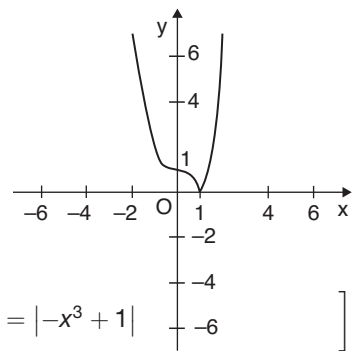
$$f(x) = |x^3|$$



$$f(x) = (x+1)^3 - 2$$

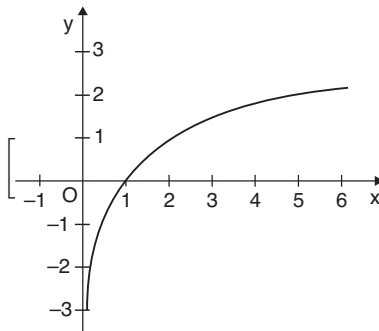


$$f(x) = -x^3 - 1$$

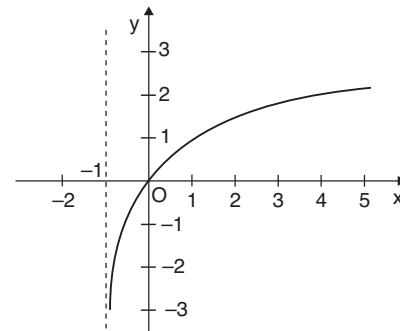


$$f(x) = |-x^3 + 1|$$

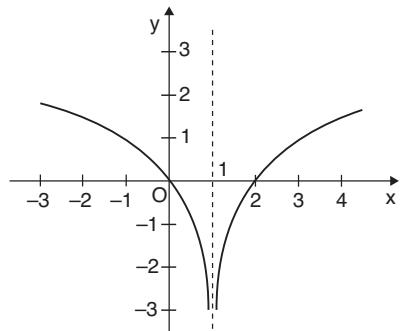
12. pag. 36



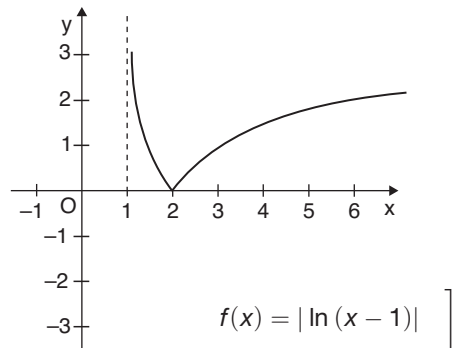
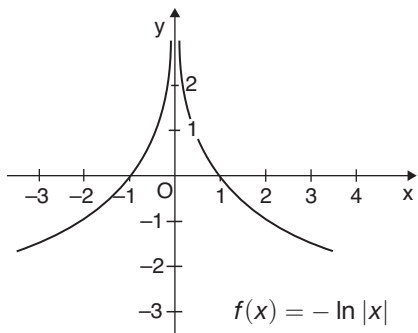
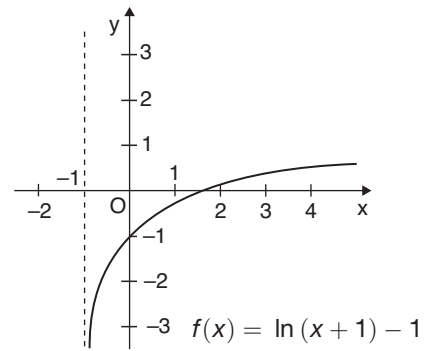
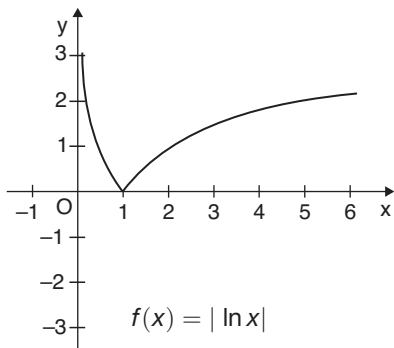
$$f(x) = \ln x$$



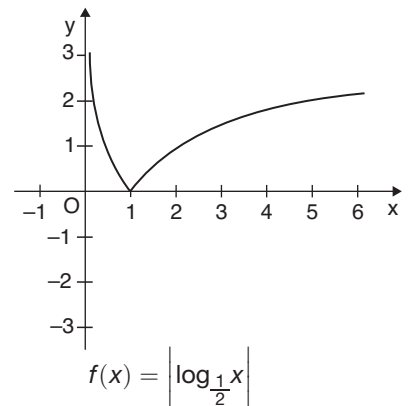
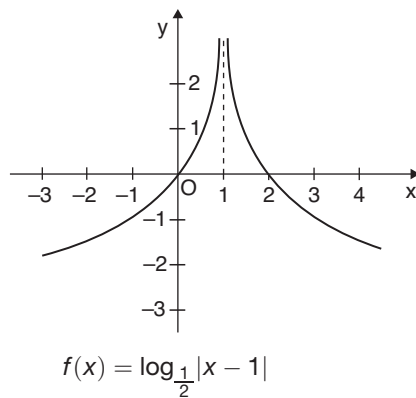
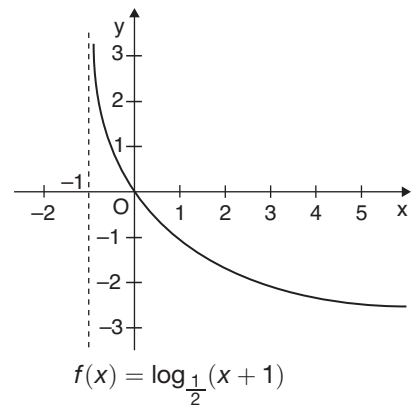
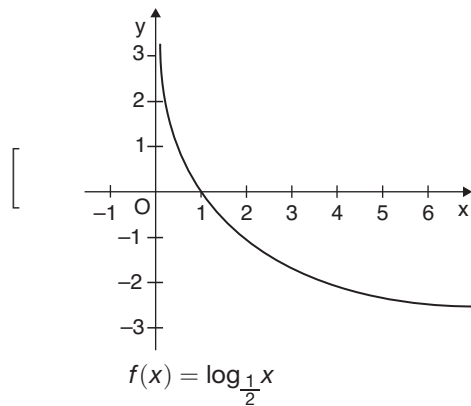
$$f(x) = \ln(x+1)$$

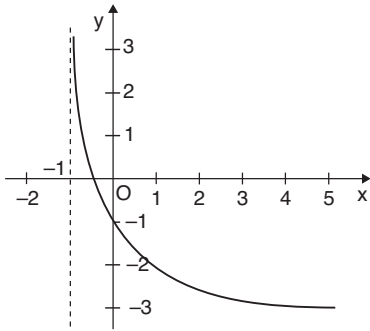


$$f(x) = \ln|x-1|$$

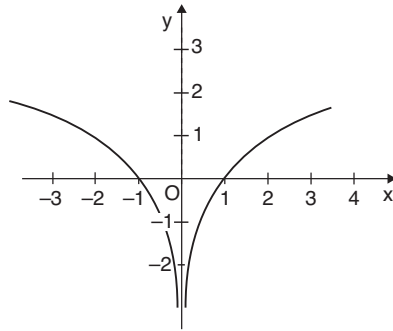


13. pag. 36

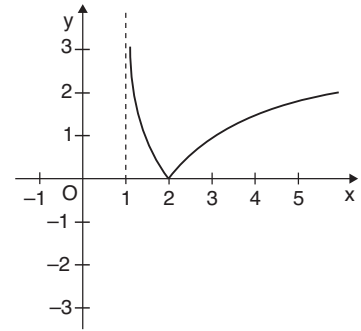




$$f(x) = \log_{\frac{1}{2}}(x+1) - 1$$

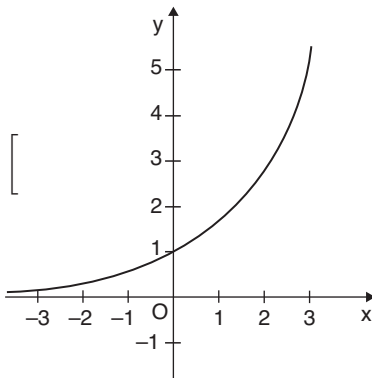


$$f(x) = -\log_{\frac{1}{2}}|x|$$

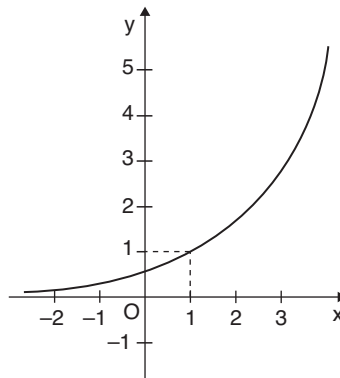


$$f(x) = \left| \log_{\frac{1}{2}}(x-1) \right|$$

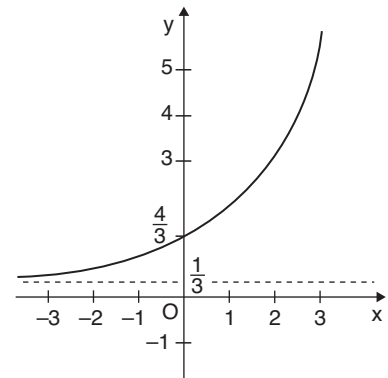
14. pag. 36



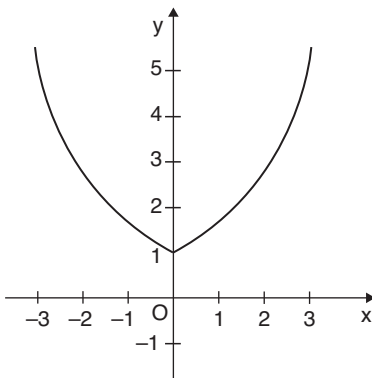
$$f(x) = 2^x$$



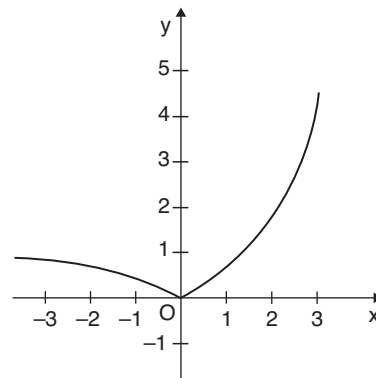
$$f(x) = 2^{(x-1)}$$



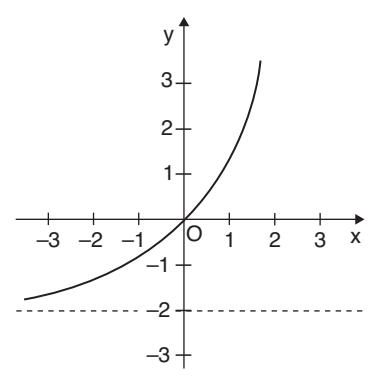
$$f(x) = \frac{1}{3} + 2^x$$



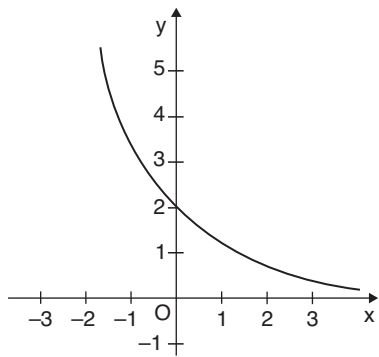
$$f(x) = 2^{|x|}$$



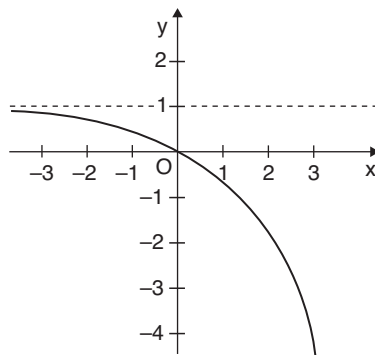
$$f(x) = |2^x - 1|$$



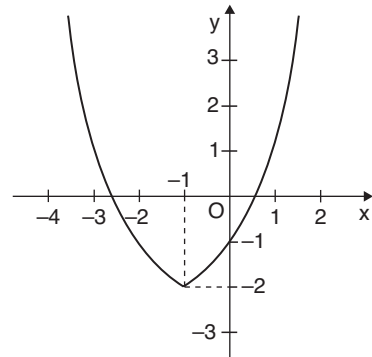
$$f(x) = 2^{(x+1)} - 2$$



$$f(x) = 2^{-(x+1)}$$

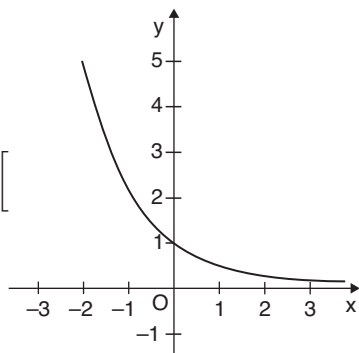


$$f(x) = 1 - 2^x$$

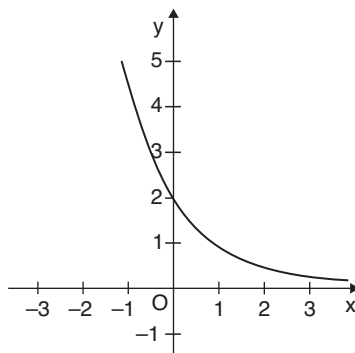


$$f(x) = 2^{|x+1|} - 3$$

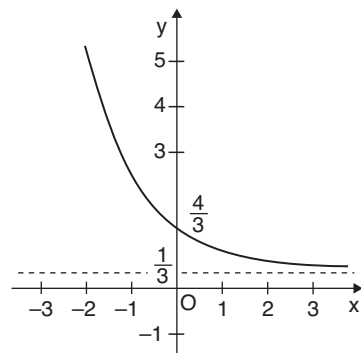
15. pag. 36



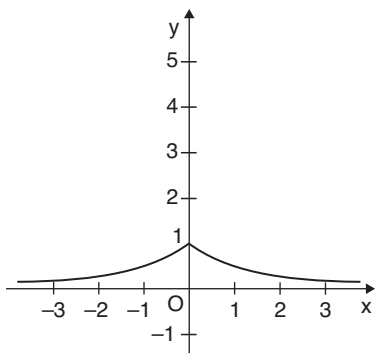
$$f(x) = \left(\frac{1}{2}\right)^x$$



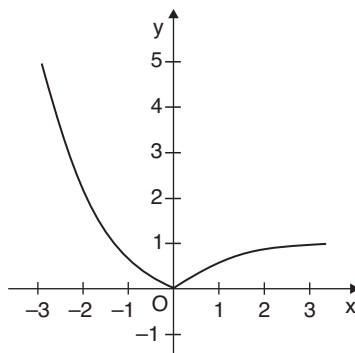
$$f(x) = \left(\frac{1}{2}\right)^{x-1}$$



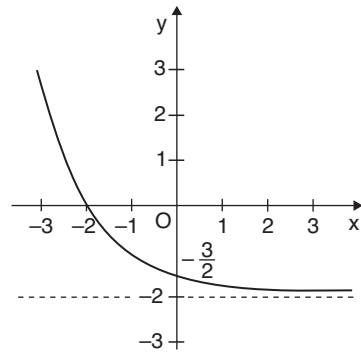
$$f(x) = \frac{1}{3} + \left(\frac{1}{2}\right)^x$$



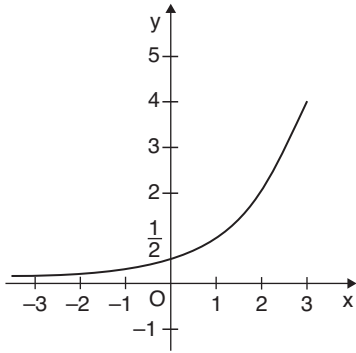
$$f(x) = \left(\frac{1}{2}\right)^{|x|}$$



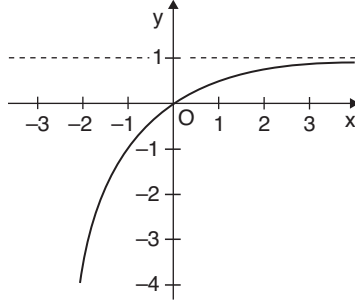
$$f(x) = \left| \left(\frac{1}{2}\right)^x - 1 \right|$$



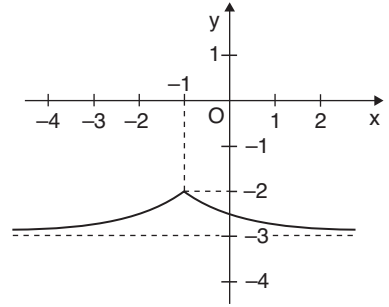
$$f(x) = \left(\frac{1}{2}\right)^{x+1} - 2$$



$$f(x) = \left(\frac{1}{2}\right)^{-x+1}$$

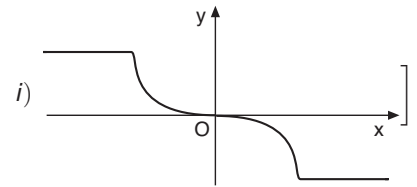
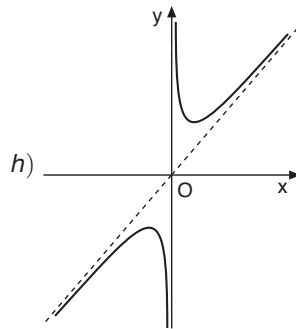
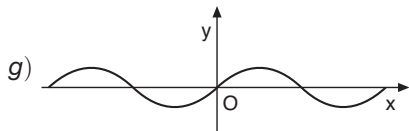
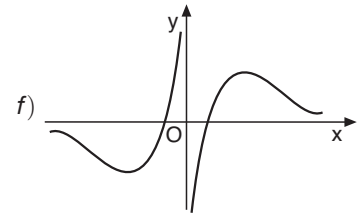
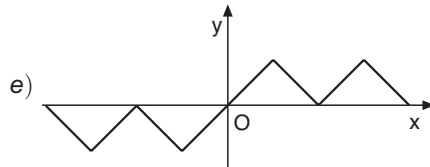
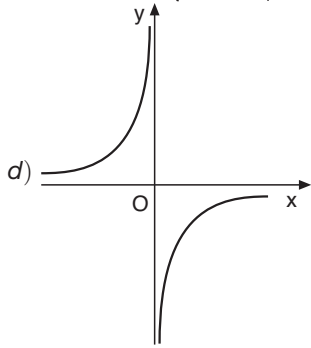
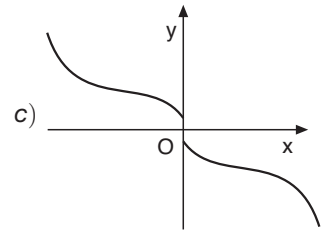
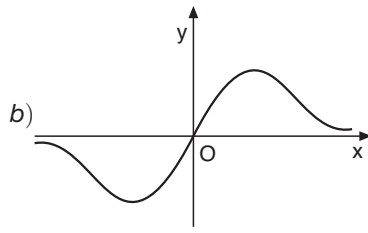
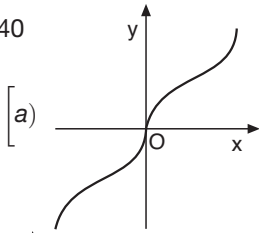


$$f(x) = 1 - \left(\frac{1}{2}\right)^x$$

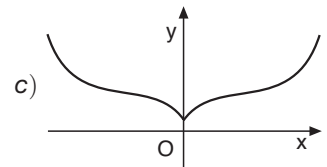
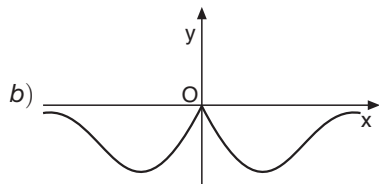
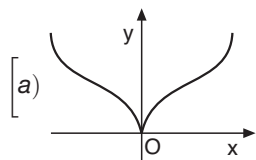


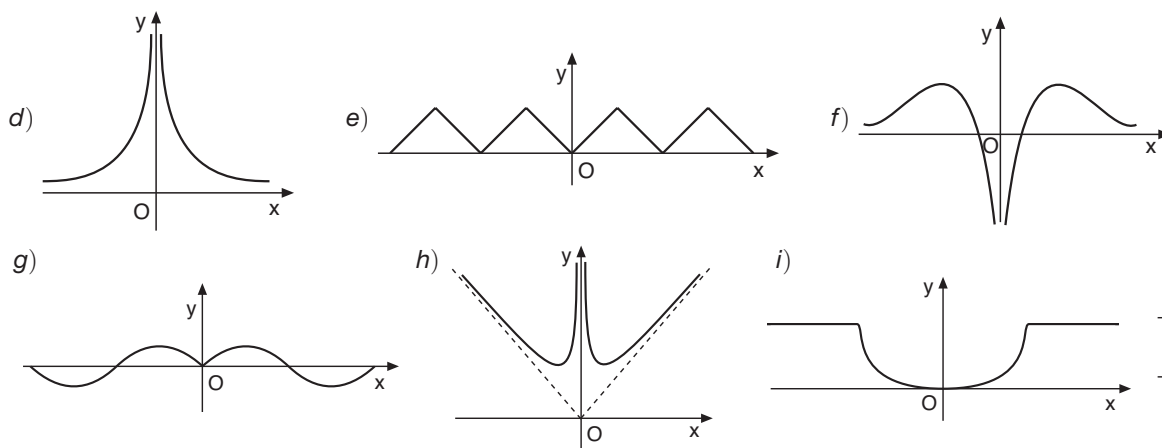
$$f(x) = \left(\frac{1}{2}\right)^{|x+1|} - 3$$

25. pag. 40



26. pag. 40





48. pag. 48 [a) $D = \mathbb{R}$; b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $(-\infty, -1]$ e in: $[-1, 0]$ e in: $[0, 1]$ e in: $[1, +\infty)$; d) $f(D) = (-\infty, 2]$; e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = 2 = M$, $x_M = 1$; $\inf f(x) = -\infty$, $\nexists m$;
 g) asse x : $P_1(0, 0) \vee P_2\left(\frac{9}{2}, 0\right)$, asse y : $P_1(0, 0)$;
 h) $f(x) > 0$: $(-\infty, 0) \cup \left(0, \frac{9}{2}\right)$, $f(x) < 0$: $\left(\frac{9}{2}, +\infty\right)$;
 i) $f(x)$ crescente in: $(-\infty, -1]$ e in: $[0, 1]$, $f(x)$ decrescente in: $[-1, 0]$ e in: $[1, +\infty)$]

49. pag. 48 [a) $D = \mathbb{R}$; b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $(-\infty, 1]$ e in: $[1, +\infty)$;
 d) $f(D) = \left[-\frac{1}{2}, +\infty\right)$; e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$;
 $\inf f(x) = -\frac{1}{2} = m$, $x_m = 1$; g) asse x : $P_1(0, 0) \vee P_2\left(\frac{4}{3}, 0\right)$, asse y : $P_1(0, 0)$;
 h) $f(x) > 0$: $(-\infty, 0) \cup \left(\frac{4}{3}, +\infty\right)$, $f(x) < 0$: $\left(0, \frac{4}{3}\right)$;
 i) $f(x)$ crescente in: $[1, +\infty)$, $f(x)$ decrescente in: $(-\infty, 1]$]

50. pag. 48 [a) $D = \mathbb{R}$; b) funzione pari; c) $f(x)$ iniettiva in: $(-\infty, -1]$ e in: $[-1, 0]$ e in: $[0, 1]$ e in: $[1, +\infty)$;
 d) $f(D) = \left[-\frac{5}{2}, +\infty\right)$; e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$;
 $\inf f(x) = -\frac{5}{2} = m$, $x_m = -1 \vee x_m = 1$;
 g) asse x : $P_1\left(-\frac{4}{3}, 0\right) \vee P_2\left(-\frac{1}{2}, 0\right) \vee P_3\left(\frac{1}{2}, 0\right) \vee P_4\left(\frac{4}{3}, 0\right)$, asse y : $Q_1(0, 4)$;
 h) $f(x) > 0$: $(-\infty, -\frac{4}{3}) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \cup \left(\frac{4}{3}, +\infty\right)$, $f(x) < 0$: $\left(-\frac{4}{3}, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{4}{3}\right)$;
 i) $f(x)$ crescente in: $[-1, 0]$ e in: $[1, +\infty)$, $f(x)$ decrescente in: $(-\infty, -1]$ e in: $[0, 1]$]

51. pag. 48 [a) $D = \left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 2\right) \cup (2, +\infty)$; b) funzione né pari, né dispari;

- c) $f(x)$ iniettiva in: $(-\infty, -1]$ e in: $\left[-1, \frac{1}{2}\right)$ e in: $\left(\frac{1}{2}, 1\right]$ e in: $[1, 2)$ e in: $(2, +\infty)$;
 d) $f(D) = (-\infty, -2] \cup \left[\frac{2}{9}, +\infty\right)$; e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$;
 f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = -\infty$, $\nexists m$; g) asse x : nessuna intersezione, asse y : $Q_1\left(0, \frac{1}{2}\right)$;
 h) $f(x) > 0$: $\left(-\infty, \frac{1}{2}\right) \cup (2, +\infty)$, $f(x) < 0$: $\left(\frac{1}{2}, 2\right)$; i) $f(x)$ crescente in: $\left[-1, \frac{1}{2}\right)$ e in: $\left(\frac{1}{2}, 1\right]$,
 $f(x)$ decrescente in: $(-\infty, -1]$ e in: $[1, 2)$ e in: $(2, +\infty)$

52. pag. 48

- In relazione all'esercizio 44 a) pag. 47, si ha:
 b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $\left(-\infty, -\frac{3}{2}\right]$ e in: $\left[-\frac{3}{2}, \frac{3}{2}\right]$ e in: $\left[\frac{3}{2}, +\infty\right)$;
 e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = -1 = m$, $x_m = -\frac{3}{2}$;
 g) asse x : $P_1(-2, 0) \vee P_2(0, 0)$, asse y : $P_2(0, 0)$; h) $f(x) > 0$: $(-\infty, -2) \cup (0, +\infty)$, $f(x) < 0$: $(-2, 0)$;
 i) $f(x)$ crescente in: $\left[-\frac{3}{2}, \frac{3}{2}\right]$, $f(x)$ decrescente in: $\left(-\infty, -\frac{3}{2}\right]$ e in: $\left[\frac{3}{2}, +\infty\right)$.

In relazione all'esercizio 44 b) pag. 47, si ha:

- b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $(-\infty, 0]$; e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$;
 f) $\sup f(x) = 2 = M$, $x_M = \{x \in \mathbb{R} | x \geq 0\}$; $\inf f(x) = -\infty$, $\nexists m$;
 g) asse x : $P_1(-2, 0)$, asse y : $Q_1(0, 2)$; h) $f(x) > 0$: $(-2, +\infty)$, $f(x) < 0$: $(-\infty, -2)$;
 i) $f(x)$ crescente in: $(-\infty, 0]$, $f(x)$ non decrescente in D .

In relazione all'esercizio 45 a) pag. 47, si ha:

- b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $[-7, -4]$ e in: $[-4, 0)$ e in: $(0, 2]$ e in: $[2, 6]$;
 e) $f(x)$ è suriettiva poiché $f(D) = \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = -\infty$, $\nexists m$;
 g) asse x : $P_1(-7, 0) \vee P_2\left(-\frac{5}{3}, 0\right) \vee P_3(1, 0) \vee P_4(5, 0)$, asse y : nessuna intersezione;
 h) $f(x) > 0$: $\left(-7, -\frac{5}{3}\right) \cup (0, 1) \cup (5, 6]$, $f(x) < 0$: $\left(-\frac{5}{3}, 0\right) \cup (1, 5)$;
 i) $f(x)$ crescente in: $[-7, -4]$ e in: $[2, 6]$, $f(x)$ decrescente in: $[-4, 0)$ e in: $(0, 2]$.

In relazione all'esercizio 45 b) pag. 47, si ha:

- b) funzione né pari, né dispari; c) $f(x)$ iniettiva in D ; e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$;
 f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = 0$, $\nexists m$; g) asse x : nessuna intersezione, asse y : nessuna intersezione;
 h) $f(x) > 0$: D , $f(x) < 0$: \emptyset ; i) $f(x)$ crescente in D .

In relazione all'esercizio 46 a) pag. 47, si ha:

- b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $[-5, -1)$ e in: $[-1, 4)$;
 e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = -4 = m$, $x_m = -1$;
 g) asse x : $P_1(-5, 0)$, asse y : $Q_1(0, -2)$; h) $f(x) > 0$: $(-5, -1)$, $f(x) < 0$: $[-1, 4)$;
 i) $f(x)$ crescente in: $[-5, -1)$ e in: $[-1, 4)$, $f(x)$ decrescente in nessun sottoinsieme di D .

In relazione all'esercizio 46 b) pag. 47, si ha:

- b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $(-\infty, -3]$ e in: $[-3, 0]$ e in: $[0, 1]$ e in: $[1, 2)$;
 e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = -\frac{3}{2} = m$, $x_m = 1$;

g) asse x : $P_1\left(\frac{1}{2}, 0\right) \vee P_2\left(\frac{3}{2}, 0\right)$, asse y : $Q_1(0, 3)$; h) $f(x) > 0$: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{3}{2}, 2\right)$, $f(x) < 0$: $\left(\frac{1}{2}, \frac{3}{2}\right)$;
 i) $f(x)$ crescente in: $[-3, 0]$ e in: $[1, 2)$, $f(x)$ decrescente in: $(-\infty, -3]$ e in: $[0, 1]$.

In relazione all'esercizio 47 a) pag. 47, si ha:

b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $(-\infty, -3]$ e in: $[-3, -1]$ e in: $[-1, 0)$ e in: $(0, +\infty)$;
 e) $f(x)$ non è suriettiva poiché $f(D) \subset \mathbb{R}$; f) $\sup f(x) = 2$, $\nexists M$; $\inf f(x) = -\infty$, $\nexists m$;

g) asse x : $P_1(-4, 0) \vee P_2(-2, 0) \vee P_3\left(-\frac{1}{2}, 0\right)$, asse y : nessuna intersezione;

h) $f(x) > 0$: $(-\infty, -4) \cup \left(-2, -\frac{1}{2}\right) \cup (0, +\infty)$, $f(x) < 0$: $(-4, -2) \cup \left(-\frac{1}{2}, 0\right)$;

i) $f(x)$ crescente in: $[-3, -1]$ e in: $(0, +\infty)$, $f(x)$ decrescente in: $(-\infty, -3]$ e in: $[-1, 0)$.

In relazione all'esercizio 47 b) pag. 47, si ha:

b) funzione né pari, né dispari; c) $f(x)$ iniettiva in: $\left[-\frac{7}{3}, 1\right]$ e in: $[1, 3)$ e in: $(3, +\infty)$;

e) $f(x)$ è suriettiva poiché $f(D) = \mathbb{R}$; f) $\sup f(x) = +\infty$, $\nexists M$; $\inf f(x) = +\infty$, $\nexists m$;

g) asse x : $P_1(0, 0) \vee P_2(2, 0)$, asse y : $P_1(0, 0)$; h) $f(x) > 0$: $\left(-\frac{7}{3}, 0\right) \cup (2, 3)$, $f(x) < 0$: $(0, 2) \cup (3, +\infty)$;

i) $f(x)$ crescente in: $[1, 3)$ e in: $(3, +\infty)$, $f(x)$ decrescente in: $\left(-\frac{7}{3}, 1\right]$

53. pag. 49

[Si ha: $h(x) = (\sin x)^2 = \sin^2 x$, $k(x) = \sin(x^2)$;
 $h(x) = \ln(x^2 - 1)$, $k(x) = (\ln x)^2 - 1 = \ln^2 x - 1$; $h(x) = 2^{2x}$, $k(x) = 2^{x^2}$]

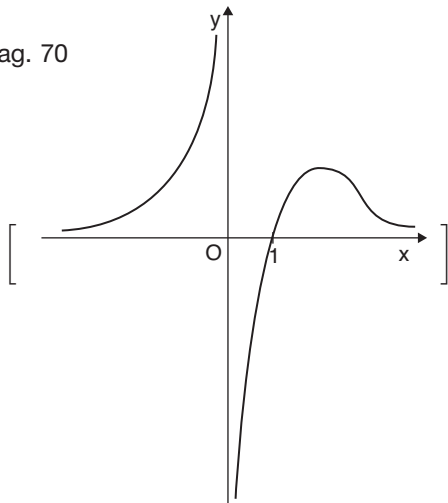
54. pag. 49

[Si ha: $h(x) = \sin(3x + 1)$, $k(x) = 3 \sin x + 1$;
 $h(x) = \sqrt{\ln x}$, $k(x) = \ln \sqrt{x} = \frac{1}{2} \ln x$; $h(x) = \frac{1}{x^2 - x + 1}$, $k(x) = \frac{1}{x^2} - \frac{1}{x} + 1$]

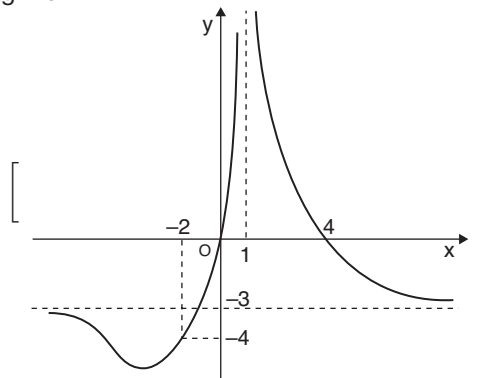
55. pag. 49

[Si ha: $h(x) = \log \frac{1+x}{1-x}$, $k(x) = \frac{1+\log x}{1-\log x}$; $h(x) = e^{\sin x} + 1$, $k(x) = \sin e^x + 1$;
 $h(x) = 4x^2 + 2x - 6$, $k(x) = 2x^2 - 10x + 3$]

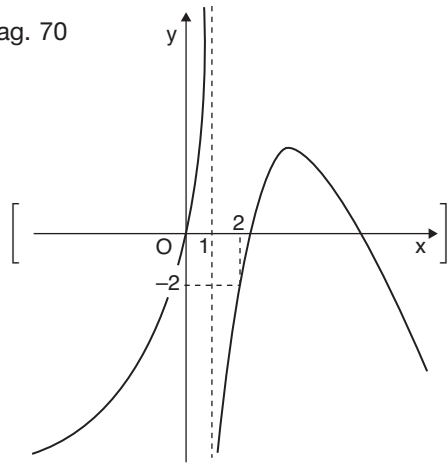
20. pag. 70



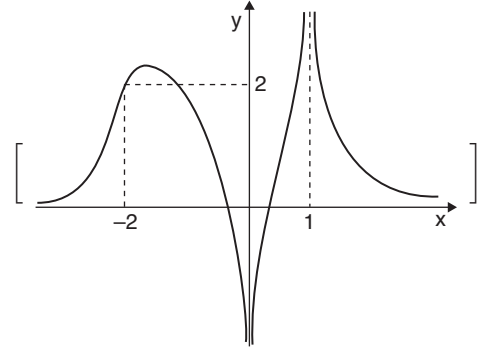
22. pag. 70



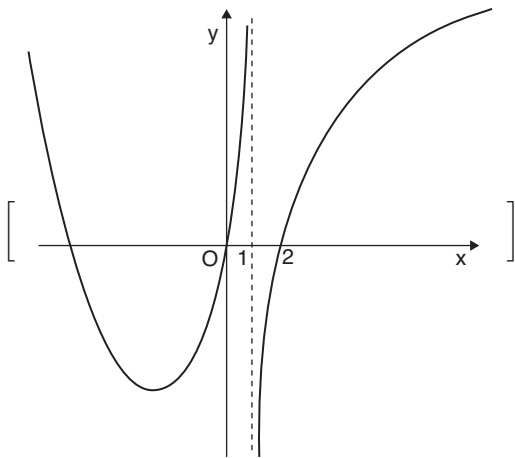
24. pag. 70



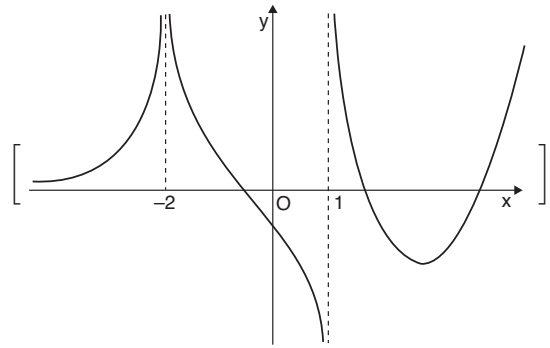
26. pag. 70



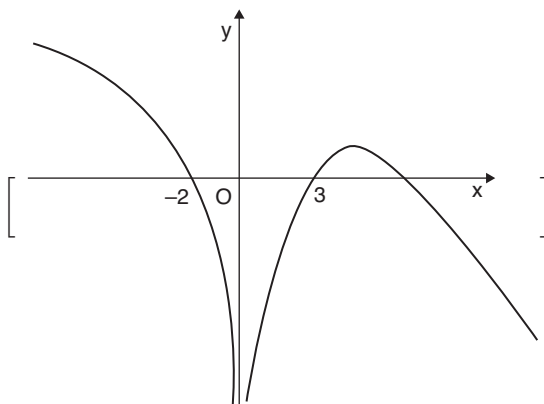
27. pag. 70



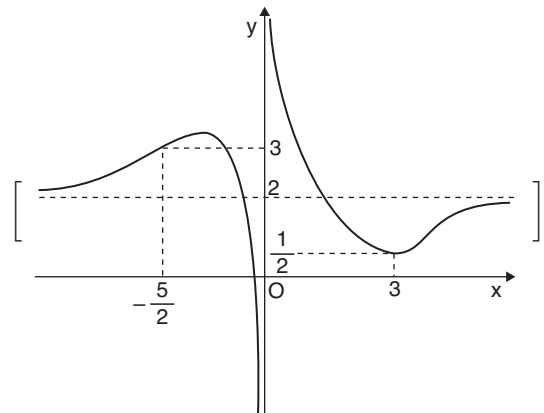
28. pag. 70



29. pag. 70



30. pag. 70



152. pag. 142

 $[D = \mathbb{R}; \text{continua e derivabile in tutto il dominio } D]$

153. pag. 142

 $[D = \mathbb{R}; \text{continua e derivabile in tutto il dominio } D]$

154. pag. 142

 $[D = \mathbb{R}; \text{continua in } D - \{1\}; x = 1 \text{ punto di discontinuità di prima specie (ricordiamo che: nei punti in cui una funzione è definita ma non è continua, non può essere derivabile); derivabile in } D - \{1\}]$

155. pag. 142

 $[D = [-2, 2]; \text{continua in tutto il dominio } D; \text{derivabile in } D - \{-1, 0, 1\}; x = -1 \vee x = 0 \vee x = 1: \text{punti angolosi}]$

156. pag. 142

 $[D = \mathbb{R}^+; \text{continua in tutto il dominio } D; \text{derivabile in } D - \{3\}; x = 3 \text{ punto angoloso}]$

157. pag. 142

 $[D = \mathbb{R}; \text{continua in tutto il dominio } D; \text{derivabile in } D - \{0\}; x = 0 \text{ punto di cuspidè}]$

158. pag. 142

 $[D = \mathbb{R}; \text{continua in tutto il dominio } D; \text{derivabile in } D - \{-1\}; x = -1 \text{ punto angoloso}]$

159. pag. 142

 $[D = \mathbb{R} - \{2\}; \text{continua in tutto il dominio } D; x = 2 \text{ punto di discontinuità di terza specie; derivabile in } D - \{-2\}; x = -2 \text{ punto di flesso a tangente verticale}]$

160. pag. 143

 $[D = [-4, -2) \cup (-2, 2) \cup (2, 4]; \text{continua in tutto il dominio } D; x = -2 \vee x = 2 \text{ punti di discontinuità di seconda specie; derivabile in tutto il dominio } D]$

161. pag. 143

 $[D = \mathbb{R} - \{1\}; \text{continua in tutto il dominio } D; x = 1 \text{ punto di discontinuità di seconda specie; derivabile in tutto il dominio } D]$

Formulario

FORMULE DI GEOMETRIA ANALITICA PIANA

► Coordinate cartesiane

- **Distanza d fra due punti $P_1(x_1, y_1)$ e $P_2(x_2, y_2)$:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **Coordinate del punto medio M del segmento di estremi $P_1(x_1, y_1)$ e $P_2(x_2, y_2)$:**

$$x_M = \frac{x_1 + x_2}{2}, \quad y_M = \frac{y_1 + y_2}{2}$$

- **Coordinate del baricentro G del triangolo di vertici $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ e $P_3(x_3, y_3)$:**

$$x_G = \frac{x_1 + x_2 + x_3}{3}, \quad y_G = \frac{y_1 + y_2 + y_3}{3}$$

► Retta

- a) **equazione generale:** $ax + by + c = 0, (a^2 + b^2 \neq 0)$
- b) **equazione esplicita:** $y = mx + q,$
- c) **equazione segmentaria:** $\frac{x}{p} + \frac{y}{q} = 1.$
- **Coefficiente angolare:**

$$m = -\frac{a}{b} = \frac{y_2 - y_1}{x_2 - x_1} = \operatorname{tg} \alpha.$$

- **Retta passante per due punti $P_1(x_1, y_1)$, $P_2(x_2, y_2)$:**

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

- **Date due rette r, s di equazione generali:** $ax + by + c = 0, \quad a_1x + b_1x + c_1 = 0,$
o di equazione esplicita: $y = mx + q, \quad y = m_1x + q_1 :$

Le due rette sono **parallele** se e soltanto se:

(1) $ab_1 = a_1b$ oppure: $m = m_1$

Le due rette sono **perpendicolari** se e soltanto se:

(2) $aa_1 + bb_1 = 0$ oppure: $mm_1 = -1$

- La retta **parallela** ad r e passante per $P_1(x_1, y_1)$ ha equazione:

$$(3) \quad \boxed{a(x - x_1) + b(y - y_1) = 0} \quad \text{oppure:} \quad \boxed{y - y_1 = m(x - x_1)}$$

- La retta **perpendicolare** ad r e passante per $P_1(x_1, y_1)$ ha equazione:

$$(4) \quad \boxed{b(x - x_1) - a(y - y_1) = 0} \quad \text{oppure:} \quad \boxed{y - y_1 = -\frac{1}{m}(x - x_1)}$$

- La **distanza** (assoluta) d del punto $P_1(x_1, y_1)$ dalla retta r è data da:

$$(5) \quad \boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}} \quad \text{oppure:} \quad \boxed{d = \frac{|mx_1 - y_1 + q|}{\sqrt{m^2 + 1}}}$$

Osservazione

Le formule (1), (2), (3), (4) scritte a sinistra sono valide *qualunque* siano le rette date, mentre quelle di destra esigono che nessuna delle rette in questione risulti parallela all'asse y .

- **Area del triangolo** di vertici $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, $P_3(x_3, y_3)$:

$$A = \pm \frac{1}{2} \begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ x_2 - x_1 & y_2 - y_1 \end{vmatrix}, \quad (\text{si sceglie il segno, } + \text{ o } -, \text{ per il quale risulta: } A \geq 0)$$

- **Bisettrici degli angoli** formati dalle due rette: $ax + by + c = 0$ e $a_1x + b_1y + c_1 = 0$:

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}.$$

- L'**angolo** α formato da due rette **non perpendicolari** e di equazioni $y = mx + q$ e $y = m_1x + q_1$ è dato da:

$$\text{tg } \alpha = \frac{m_1 - m}{1 + mm_1}.$$

► Circonferenza

Circonferenza di centro $C(\alpha, \beta)$ e raggio r :

$$\boxed{(x - \alpha)^2 + (y - \beta)^2 = r^2, \quad \text{oppure:} \quad x^2 + y^2 + ax + by + c = 0}$$

con:

$$\alpha = -\frac{a}{2}, \quad \beta = -\frac{b}{2}, \quad r = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c}, \quad \text{con} \quad \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c \geq 0.$$

- Se $P_1(x_1, y_1)$ sta sulla circonferenza $T: x^2 + y^2 + ax + by + c = 0$, l'equazione della tangente in P_1 a T ha equazione:

$$x_1x + y_1y + a\frac{x+x_1}{2} + b\frac{y+y_1}{2} + c = 0.$$

► Ellisse

Ellisse di centro O e fuochi $F'(-c, 0)$, $F(c, 0)$:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{con:} \quad b^2 = a^2 - c^2. \quad \text{Eccentricità: } e = \frac{c}{a} < 1.$$

► Iperbole

Iperbole di centro O e fuochi $F'(-c, 0)$, $F(c, 0)$:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad \text{con:} \quad b^2 = c^2 - a^2. \quad \text{Eccentricità: } e = \frac{c}{a} > 1.$$

$$\text{Asintoti:} \quad y = \frac{b}{a}x \quad \text{e} \quad y = -\frac{b}{a}x,$$

- **Iperbole equilatera:** $x^2 - y^2 = a^2$. Asintoti: $y = \pm x$.
- **Iperbole equilatera riferita agli asintoti:** $xy = k$.
- **Funzione omografica:** $y = \frac{ax + b}{cx + d}$.

Se $c \neq 0$ e $ad \neq bc$ essa è rappresentata, in coordinate cartesiane ortogonali, da un'iperbole equilatera che ha per asintoti le rette di equazioni:

$$x = -\frac{d}{c} \quad \text{e} \quad y = \frac{a}{c}.$$

► Parabola

Parabola, ad asse parallelo all'asse y , di fuoco F e direttrice d :

$$y = ax^2 + by + c.$$

In una tale parabola:

- il **vertice** è il punto di coordinate: $\boxed{-\frac{b}{2a}, -\frac{\Delta}{4a}}$ (con $\Delta = b^2 - 4ac$),

l'**asse di simmetria** è la retta parallela all'asse y , di equazione: $\boxed{x = -\frac{b}{2a}}$

- il **fuoco** è il punto di coordinate: $\boxed{-\frac{b}{2a}, \frac{1}{4a} - \frac{\Delta}{4a}}$

e la **direttrice** è la retta parallela all'asse x , di equazione: $\boxed{y = -\frac{1}{4a} - \frac{\Delta}{4a}}$

- In particolare, se il vertice è nell'origine delle coordinate, la parabola ha equazione $y = ax^2$ e si ha:

$$\text{vertice: } V(0, 0); \quad \text{asse: } x = 0; \quad F\left(0, \frac{1}{4a}\right); \quad \text{direttrice: } y = -\frac{1}{4a}.$$

Analoghi risultati, se si assume l'asse y parallelo alla direttrice.

► Trasformazione delle coordinate

- **Traslazione** Se $P(x, y)$ nel sistema Oxy e $P(X, Y)$ nel sistema $O'XY$, $O'(a, b)$ nel sistema Oxy e gli assi sono rispettivamente **paralleli ed equiversi**:

$$\begin{cases} x = X + a \\ y = Y + b \end{cases} \quad \text{oppure:} \quad \begin{cases} X = x - a \\ Y = y - b. \end{cases}$$

- **Rotazione** Se $P(x, y)$ nel sistema Oxy e $P(X, Y)$ nel sistema OXY e $\alpha = \angle XO'X$:

$$\begin{cases} x = X \cos \alpha - Y \sin \alpha \\ y = X \sin \alpha + Y \cos \alpha, \end{cases} \quad \text{oppure:} \quad \begin{cases} X = x \cos \alpha + y \sin \alpha \\ Y = -x \sin \alpha + y \cos \alpha. \end{cases}$$

- **Rototraslazione** Se $P(x, y)$ nel sistema Oxy , $P(X, Y)$ nel sistema $O'XY$, $\alpha = \angle XO'X$ e $O'(a, b)$ nel sistema Oxy :

$$\begin{cases} x = a + X \cos \alpha - Y \sin \alpha \\ y = b + X \sin \alpha + Y \cos \alpha, \end{cases} \quad \text{oppure:} \quad \begin{cases} X = (x - a) \cos \alpha + (y - b) \sin \alpha \\ Y = -(x - a) \sin \alpha + (y - b) \cos \alpha. \end{cases}$$

- Un punto P del piano, oltre che dalle coordinate cartesiane x, y , può essere individuato dalle **coordinate polari**, legate alle precedenti dalle relazioni:

$$\begin{cases} x = \rho \cos \vartheta \\ y = \rho \sin \vartheta \end{cases} \quad \text{oppure:} \quad \begin{cases} \rho = \sqrt{x^2 + y^2} \\ \text{tg } \vartheta = \frac{y}{x}. \end{cases}$$

ESPONENZIALI E LOGARITMI

- Se $a > 0$, $b > 0$ e $a \neq 1$, l'equazione: $a^x = b$ ammette una e una sola soluzione.
 $a^x = b \Leftrightarrow x = \log_a b \Rightarrow a^{\log_a b} = b.$

• **Teoremi sui logaritmi** (salve le ipotesi su a, b, c, n, N):

- | | |
|--|---|
| 1) $\log_a(bc) = \log_a b + \log_a c;$ | 2) $\log_a b^c = c \log_a b;$ |
| 3) $\log_a \frac{b}{c} = \log_a b - \log_a c;$ | 4) $\log_a \sqrt[n]{b} = \frac{1}{n} \log_a b;$ |
| 5) $\log_b N = \frac{\log_a N}{\log_a b};$ | 6) $\log_a b \cdot \log_b a = 1;$ |
| 7) $\log_{a^n} b^m = \frac{m}{n} \log_a b;$ | 8) $\log_a b = \log_{a^n} b^n.$ |

Attenzione: $\log x^2 = 2 \log |x|.$

FORMULE DI TRIGONOMETRIA

► **Espressione di tutte le funzioni goniometriche di un angolo orientato mediante una sola di esse.**

Nota	sen α	cos α	tg α	ctg α
sen α	sen α	$\sqrt{1 - \text{sen}^2 \alpha}$	$\frac{\text{sen } \alpha}{\pm \sqrt{1 - \text{sen}^2 \alpha}}$	$\pm \frac{\sqrt{1 - \text{sen}^2 \alpha}}{\text{sen } \alpha}$
cos α	$\pm \sqrt{1 - \text{cos}^2 \alpha}$	cos α	$\pm \frac{\sqrt{1 - \text{cos}^2 \alpha}}{\text{cos } \alpha}$	$\frac{\text{cos } \alpha}{\pm \sqrt{1 - \text{cos}^2 \alpha}}$
tg α	$\frac{\text{tg } \alpha}{\pm \sqrt{1 + \text{tg}^2 \alpha}}$	$\frac{1}{\pm \sqrt{1 + \text{tg}^2 \alpha}}$	tg α	$\frac{1}{\text{tg } \alpha}$
ctg α	$\frac{1}{\pm \sqrt{1 + \text{ctg}^2 \alpha}}$	$\frac{\text{ctg } \alpha}{\pm \sqrt{1 + \text{ctg}^2 \alpha}}$	$\frac{1}{\text{ctg } \alpha}$	ctg α

► **Angoli associati**

ANGOLI OPPOSTI

$$\begin{cases} \text{sen}(-\alpha) = -\text{sen } \alpha \\ \text{cos}(-\alpha) = \text{cos } \alpha \\ \text{tg}(-\alpha) = -\text{tg } \alpha \end{cases}$$

ANGOLI SUPPLEMENTARI

$$\begin{cases} \text{sen}(\pi - \alpha) = \text{sen } \alpha \\ \text{cos}(\pi - \alpha) = -\text{cos } \alpha \\ \text{tg}(\pi - \alpha) = -\text{tg } \alpha \end{cases}$$

ANGOLI CHE DIFFERISCONO DI UN ANGOLO PIATTO

$$\begin{cases} \text{sen}(\alpha + \pi) = -\text{sen } \alpha \\ \text{cos}(\alpha + \pi) = -\text{cos } \alpha \\ \text{tg}(\alpha + \pi) = \text{tg } \alpha \end{cases}$$

ANGOLI ESPLEMENTARI⁽¹⁾

$$\begin{cases} \operatorname{sen}(2\pi - \alpha) = -\operatorname{sen} \alpha \\ \operatorname{cos}(2\pi - \alpha) = \operatorname{cos} \alpha \\ \operatorname{tg}(2\pi - \alpha) = -\operatorname{tg} \alpha \end{cases}$$

ANGOLI COMPLEMENTARI

$$\begin{cases} \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{cos} \alpha \\ \operatorname{cos}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{sen} \alpha \\ \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) = \operatorname{ctg} \alpha \end{cases}$$

ANGOLI CHE DIFFERISCONO
DI UN ANGOLO RETTO

$$\begin{cases} \operatorname{sen}\left(\alpha + \frac{\pi}{2}\right) = \operatorname{cos} \alpha \\ \operatorname{cos}\left(\alpha + \frac{\pi}{2}\right) = -\operatorname{sen} \alpha \\ \operatorname{tg}\left(\alpha + \frac{\pi}{2}\right) = -\operatorname{ctg} \alpha \end{cases}$$

► Formule goniometriche

FORMULE DI ADDIZIONE

$$\begin{cases} \operatorname{sen}(\alpha + \beta) = \operatorname{sen} \alpha \operatorname{cos} \beta + \operatorname{sen} \beta \operatorname{cos} \alpha \\ \operatorname{cos}(\alpha + \beta) = \operatorname{cos} \alpha \operatorname{cos} \beta - \operatorname{sen} \alpha \operatorname{sen} \beta \\ \operatorname{tg}(\alpha + \beta) = \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{1 - \operatorname{tg} \alpha \operatorname{tg} \beta} \end{cases}$$

FORMULE DI SOTTRAZIONE

$$\begin{cases} \operatorname{sen}(\alpha - \beta) = \operatorname{sen} \alpha \operatorname{cos} \beta - \operatorname{sen} \beta \operatorname{cos} \alpha \\ \operatorname{cos}(\alpha - \beta) = \operatorname{cos} \alpha \operatorname{cos} \beta + \operatorname{sen} \alpha \operatorname{sen} \beta \\ \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \operatorname{tg} \beta} \end{cases}$$

FORMULE DI DUPLICAZIONE

$$\begin{cases} \operatorname{sen} 2\alpha = 2 \operatorname{sen} \alpha \operatorname{cos} \alpha \\ \operatorname{cos} 2\alpha = \operatorname{cos}^2 \alpha - \operatorname{sen}^2 \alpha = 1 - 2 \operatorname{sen}^2 \alpha = \\ = 2 \operatorname{cos}^2 \alpha - 1 \\ \operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} \end{cases}$$

FORMULE DI TRIPLICAZIONE

$$\begin{cases} \operatorname{sen} 3\alpha = 3 \operatorname{sen} \alpha - 4 \operatorname{sen}^3 \alpha \\ \operatorname{cos} 3\alpha = 4 \operatorname{cos}^3 \alpha - 3 \operatorname{cos} \alpha \\ \operatorname{tg} \alpha = \frac{3 \operatorname{tg} \alpha - \operatorname{tg}^3 \alpha}{1 - 3 \operatorname{tg}^2 \alpha} \end{cases}$$

FORMULE DI BISEZIONE

$$\begin{cases} \operatorname{sen} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \operatorname{cos} \alpha}{2}}, \\ \operatorname{cos} \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \operatorname{cos} \alpha}{2}}, \\ \operatorname{tg} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \operatorname{cos} \alpha}{1 + \operatorname{cos} \alpha}} = \frac{\operatorname{sen} \alpha}{1 + \operatorname{cos} \alpha} = \frac{1 - \operatorname{cos} \alpha}{\operatorname{sen} \alpha} \end{cases}$$

FORMULE DI PROSTAFERESI

$$\begin{cases} \operatorname{sen} p + \operatorname{sen} q = 2 \operatorname{sen} \frac{p+q}{2} \operatorname{cos} \frac{p-q}{2} \\ \operatorname{sen} p - \operatorname{sen} q = 2 \operatorname{cos} \frac{p+q}{2} \operatorname{sen} \frac{p-q}{2} \\ \operatorname{cos} p + \operatorname{cos} q = 2 \operatorname{cos} \frac{p+q}{2} \operatorname{cos} \frac{p-q}{2} \\ \operatorname{cos} p - \operatorname{cos} q = -2 \operatorname{sen} \frac{p+q}{2} \operatorname{sen} \frac{p-q}{2} \end{cases}$$

FORMULE DI WERNER

$$\begin{cases} \operatorname{sen} \alpha \operatorname{sen} \beta = \frac{1}{2} [\operatorname{cos}(\alpha - \beta) - \operatorname{cos}(\alpha + \beta)] \\ \operatorname{cos} \alpha \operatorname{cos} \beta = \frac{1}{2} [\operatorname{cos}(\alpha + \beta) + \operatorname{cos}(\alpha - \beta)] \\ \operatorname{sen} \alpha \operatorname{cos} \beta = \frac{1}{2} [\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta)] \end{cases}$$

ESPRESSIONE DI $\operatorname{sen} \alpha$ E $\operatorname{cos} \alpha$ IN FUNZIONE RAZIONALE DI $\operatorname{tg} \frac{\alpha}{2}$

$$\operatorname{sen} \alpha = \frac{2 \operatorname{tg} \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}; \quad \operatorname{cos} \alpha = \frac{1 - \operatorname{tg}^2 \frac{\alpha}{2}}{1 + \operatorname{tg}^2 \frac{\alpha}{2}}; \quad \text{cioè: } \operatorname{sen} \alpha = \frac{2t}{1 + t^2}; \quad \operatorname{cos} \alpha = \frac{1 - t^2}{1 + t^2}, \quad \text{con } t = \operatorname{tg} \frac{\alpha}{2}.$$

⁽¹⁾ Si noti che le formule sono ancora quelle degli angoli opposti.

► **Funzioni goniometriche di angoli notevoli**

Angolo orientato		Funzione goniometrica			
in gradi	in radianti	seno	coseno	tangente	cotangente
0°	0	0	1	0	<i>non esiste</i>
9°	$\frac{\pi}{20}$	$\frac{\sqrt{3 + \sqrt{5}} - \sqrt{5 - \sqrt{5}}}{4}$	$\frac{\sqrt{3 + \sqrt{5}} + \sqrt{5 - \sqrt{5}}}{4}$	$\frac{4 - \sqrt{10 + 2\sqrt{5}}}{\sqrt{5} - 1}$	$\frac{\sqrt{5} - 1}{4 - \sqrt{10 + 2\sqrt{5}}}$
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$2 - \sqrt{3}$	$2 + \sqrt{3}$
18°	$\frac{\pi}{10}$	$\frac{\sqrt{5} - 1}{4}$	$\sqrt{\frac{10 + 2\sqrt{5}}{4}}$	$\sqrt{\frac{5 - 2\sqrt{5}}{5}}$	$\sqrt{5 + 2\sqrt{5}}$
22°30'	$\frac{\pi}{8}$	$\frac{\sqrt{2} - \sqrt{2}}{2}$	$\frac{\sqrt{2} + \sqrt{2}}{2}$	$\sqrt{2} - 1$	$\sqrt{2} + 1$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\sqrt{3}$
36°	$\frac{\pi}{5}$	$\frac{\sqrt{10 - 2\sqrt{5}}}{4}$	$\frac{\sqrt{5} + 1}{4}$	$\sqrt{5 - 2\sqrt{5}}$	$\frac{\sqrt{25 + 10\sqrt{5}}}{5}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1	1
54°	$\frac{3\pi}{10}$	$\frac{\sqrt{5} + 1}{4}$	$\frac{\sqrt{10 - 2\sqrt{5}}}{4}$	$\frac{\sqrt{25 + 10\sqrt{5}}}{5}$	$\sqrt{5 - 2\sqrt{5}}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
72°	$\frac{3\pi}{5}$	$\frac{\sqrt{10 + 2\sqrt{5}}}{4}$	$\frac{\sqrt{5} - 1}{4}$	$\sqrt{5 + 2\sqrt{5}}$	$\frac{\sqrt{25 - 10\sqrt{5}}}{5}$
75°	$\frac{5\pi}{12}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$2 + \sqrt{3}$	$2 - \sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	<i>non esiste</i>	0

- **Alcuni grafici notevoli** pag. 23
- **Formule e regole di derivazione** pag. 114
- **Tablelle delle primitive di funzioni d'uso frequente** pag. 298

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